Measuring Fuel Consumption for an Electric Vehicle

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The United States has long used miles per gallon to measure automotive fuel consumption (called *mileage*). Europe uses liters per 100 kilometers (Canada follows Europe's lead). Is one better than the other? Of course, this is related to the general debate about the metric system vs. the US's customary units. The American system does have its advantages, and there are interesting arguments on both sides. But the issue here is not related to the units per se, but to how the units are combined.

When hybrid cars became available, there was the possibility of negative fuel consumption over a significant distance. That makes things interesting. And now with the advent of fully electric cars (I just purchased a Chevrolet Bolt) it gets even more interesting. In the USA, electric vehicle efficiency is measured in miles/kWh (a typical value is 4); in Europe and Canada it is done in kWh/(100 km) (a typical value is 15.5).

Suppose one takes a drive that starts with a steep downhill of five miles and finishes with a slight uphill of the same distance. So perhaps the battery gains 2 kWh for every downhill mile and then expends 3 kWh for each of the last five miles. What does the fuel consumption rate— the *mileage*—do? In the USA system the graph of this function is a bit of a disaster (Fig. 1).

For the first 5 miles the rate gain is constant: 2 kWh per mile. So the miles-per-kWh rate is $1/2$, but it is negative. The consumption rate is therefore $-1/2$ mile per kWh. Now, when the distance is d, between 5 and 10, the total kWh used is -10 (banked in first 5 miles) plus 3 times the miles after the end of the downhill, $d-5$. So the mileage formula is $d/(3(d-5)-10)$ or $d/(3d-25)$. This looks innocent enough until you examine the situation at $d = 25/3 = 8.33$. At that point, one has traveled 8.33 miles on 0 energy consumption, and 8.33 divided by 0 is not defined; just before 8.33 this will be a very large negative number; just after 8.33 it will be a very large finite number. As one continues on from there. the mileage rate comes down from this lofty perch until, at 10 miles, one has used 5 kWh for an overall average of 2 miles per kWh.

Note that on the graph of Figure 1, it is always better to move up, with one awkward exception. Getting 6 miles on a kWh is better than getting 4. Getting $-1/10$ is a gain of 10 kWh for every mile, which is better than $-1/2$. But 0 is not better than any negative number. So we have a discontinuity when crossing the horizontal axis. The best location on this graph is infinitesimally below 0, while the worst place is at 0.

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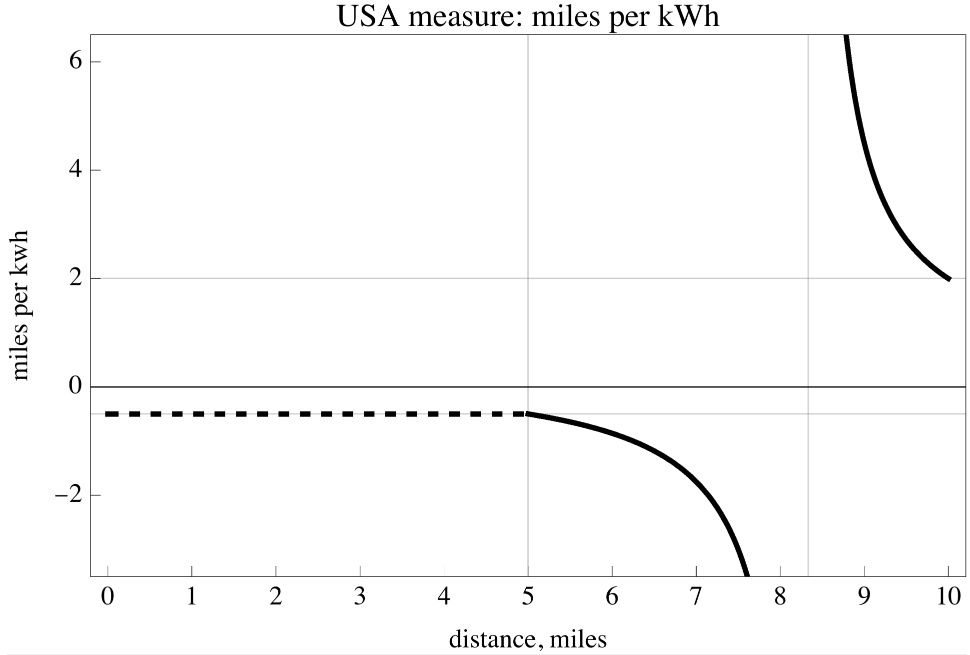

Figure 1. The fuel consumption for a drive of 5 miles downhill followed by 5 miles gentle climb, measured in miles per kWh.

In Figure 1, the dashed line refers to the steady gain of power on the downhill. At $d = 5$, one has stored 10 kWh. This means the mileage is 5 miles for −10 kwh, which is the same as −5 miles for 10 kwh; in either case it is $-1/2$. Now on the flat section one starts using energy. One has 10 kWh in the bank, but now one uses energy to move, so the rate of $-1/2$ decreases. When it becomes −3, say, that means on average up to that point one needed 3 miles to gain 1 kWh, which is worse than the early gain of a kWh in each half-mile. So when this graph decreases, so does fuel economy. But at 8.33 miles, one has used 0 kWh; the ratio there is infinite. Disaster (or, as mathematicians say, *Singularity*). At the end of the trip one has gone 10 miles and used 5 kWh, for a ratio of 2 miles per kWh.

Now in the European system it is quite different! The values are the reciprocals of the values in the American system. The curve (Fig. 2) still has a sharp corner, but that is an artifact of the too-sudden change from −2 to 4 in the simple data. The main point is that the graph is now continuous: the singularity has disappeared, to be replaced by simply a part of the graph passing through 0. And here, of course, a rising number means increasing consumption and lower fuel efficiency. The value at the end is now the reciprocal of the 2 in the American case. The continuity of this graph is much more natural, and makes the whole subject easier to grasp.

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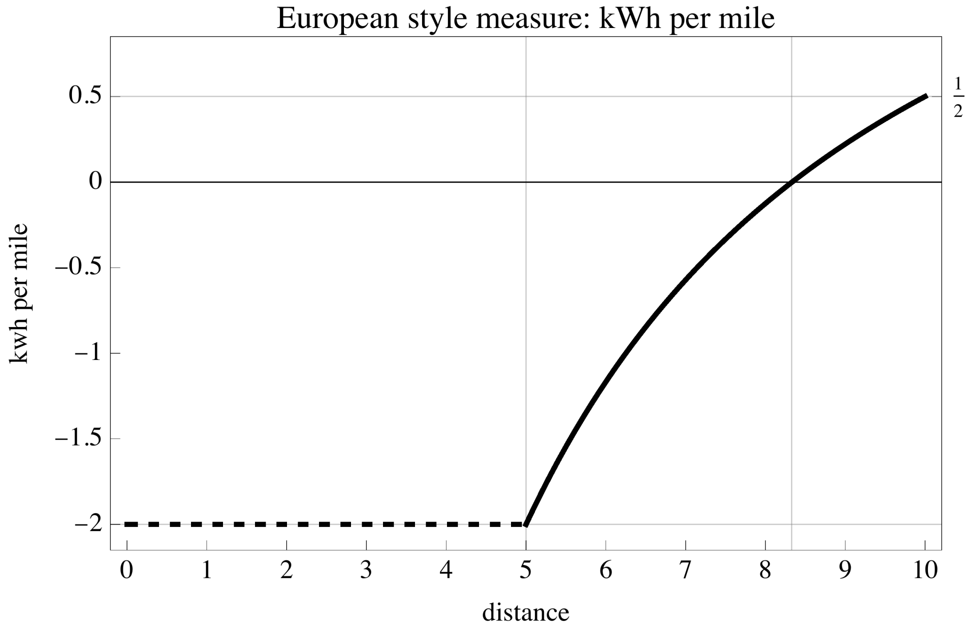

Figure 2. The fuel consumption for the same trip as in Figure 1, but measured in kWh per mile.

Where is the best place to be on this graph? Very simple: the lower the better. Using −1 kWh is better than using none, which is better than using 1. It is all transparent and easy to grasp.

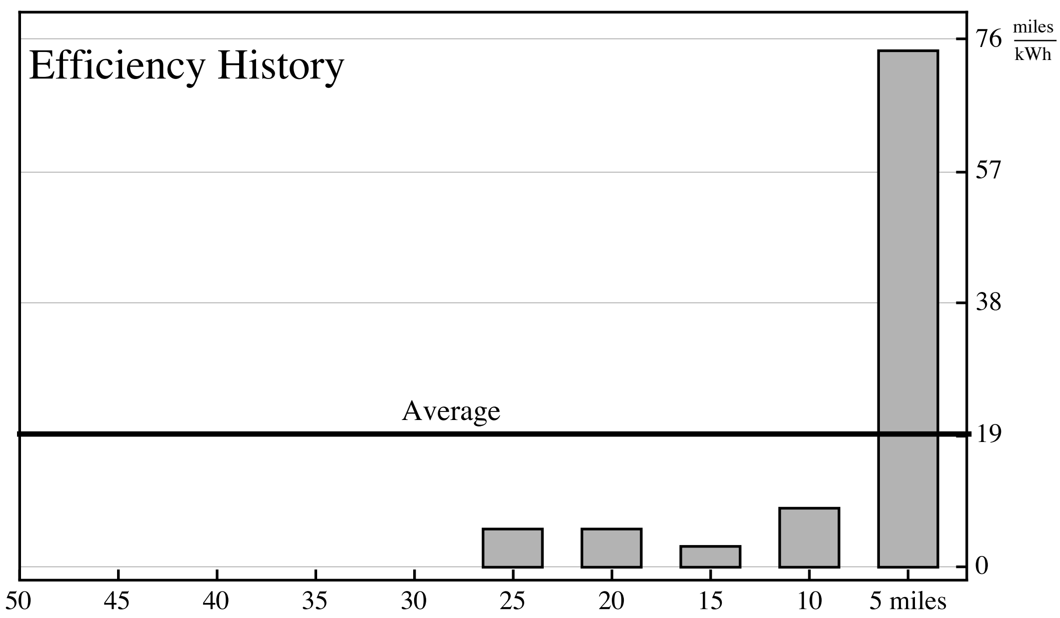

Figure 3. Actual rendition of a screen that the Bolt presents the driver in the hope of communicating the average consumption over 25 miles. The value of 19 is grossly incorrect. The correct value is 6 mi/kWh.

Now, here is an interesting and troubling aspect of the two methods. Bar charts are popular for displaying data. A screen on the Bolt can look like Figure 3. There the vertical bars for 5-mile segments give the efficiency for that segment. The numbers here are 5.5, 5.5, 3., 8.5, 74.4. The large number is because the most recent segment had a long downhill. Now a minor issue is that these values can be negative, but the screen never allows for that, replacing any negative number by 252, an upper bound. A much more serious problem concerns the horizontal line marked “Average”. The programmer here simply took the arithmetic mean of the five values, to get about 19. In fact the average efficiency is about 6. For the proper answer one simply has to take reciprocals, average, then take a final reciprocal. In short, the harmonic mean of the values gives the correct answer. And then there is the problem that the scale is badly skewed when there is a large number, so that the average line will be very hard to read.

If you are unfamiliar with how this works, consider this simple example: Imagine two bars, one at 247 and the other at 3. The average of these two is 125. But the truth is that 5/247 kWh were used on one segment and 5/3 on the other. So the total usage is 0.02 + 1.67 = 1.69 kWh. Over 10 miles, this gives a rate 10/1.69 or about 6. The difference here is enormous. The screen would say that one got 125 miles per kWh; in fact one got 6. This is almost a 2000% error.

When the system is switched to metric, the chart shown is the one in Figure 4, which is perfectly correct. Moreover, in the European system, the simple use of an average of the bar heights works perfectly. And more important, the visual impression of the bar charts allows one to estimate the average. In the American system, the bars are not really helpful. I am not suggesting that General Motors should change to the European system. We are heavily invested in the distance-per-fuel paradigm. But if the average line were in the right place, that would be an excellent start to a more accurate system.

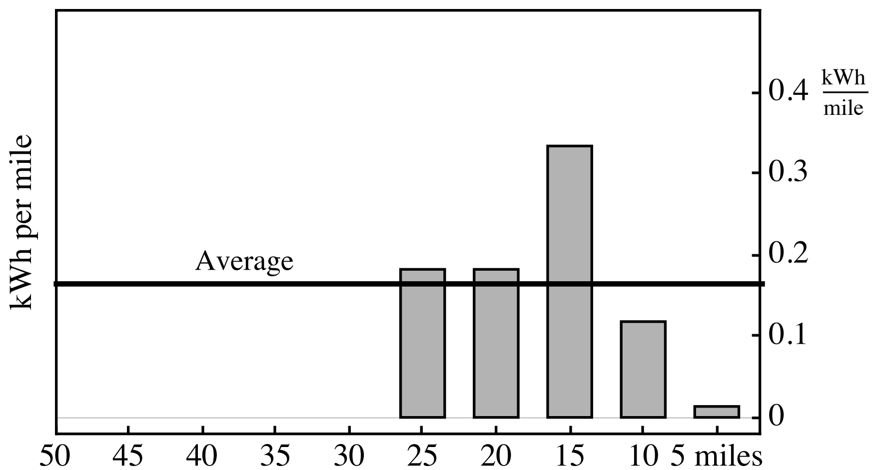

Figure 4. In the inverted system (kWh per mile) the bar heights give an immediate visual feel for the average energy consumption.

I have so far not succeeded in communicating this issue to anyone at GM who understands its importance. If any reader knows how I might do so, do tell.