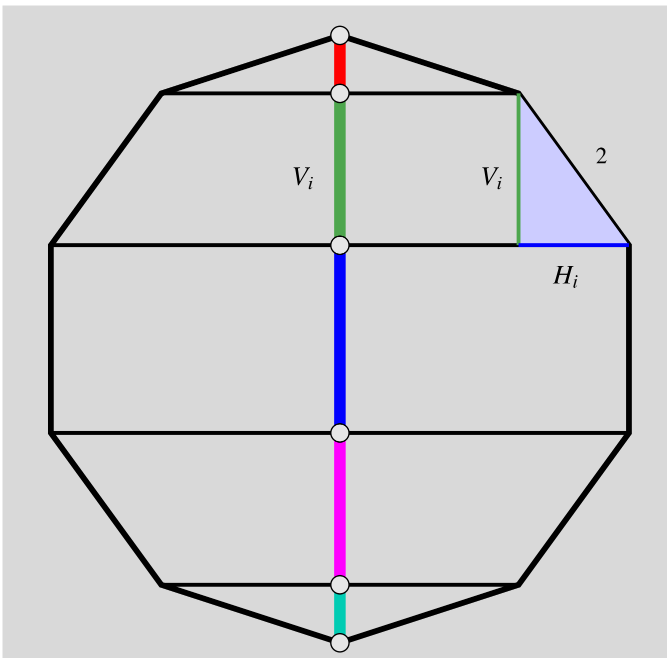
Solution to Problem 1325 by M. Elgersma and S. Wagon. The segments of the problem are the vertical components of the polygon's sides on its right half; these lengths are $V_k = 2\sin\left((2k-1)\,\frac{\pi}{n}\right)$, with $k = 1, 2, \ldots, \frac{n}{2}$ (proof uses the fact that the polygon's side rotates through $2\pi/n$ as as it moves to the next side). Similarly, the horizontal segments corresponding to the sides have lengths $H_k = 2\cos\left((2k-1)\,\frac{\pi}{n}\right)$ with same k-values.



Claim 1. $\sum\left(H_k^2 + V_k^2\right) = 2n$. Proof. The sum is $\frac{n}{2}\,2^2$ by Pythagoras.

Claim 2. $\sum H_k^2 = \sum V_k^2$. Proof. By the double-angle formula,

$$\sum H_k^2 - \sum V_k^2 = \sum\left(H_k^2 - V_k^2\right) = 2^2 \sum \cos\left((2\,k-1)\,\frac{2\pi}{n}\right).$$

The points $\left(\cos\left((2k-1)\,\frac{2\pi}{n}\right), \sin\left((2k-1)\,\frac{2\pi}{n}\right)\right)$ form a regular $\frac{n}{2}$-gon, so they sum to $(0, 0)$ and the cosines sum to 0.

The claims imply that $\sum V_k^2 = \frac{1}{2}\,2n = n$, solving the problem. □

By Pythagoras, $\sum\left(H_k^2 + V_k^2\right) = \frac{n}{2}\, 2^2 = 2n$. If $\sum H_k^2 = \sum V_k^2$, then $\sum V_k^2 = n$, solving the problem. By the double-angle formula,

$$\sum H_k^2 - \sum V_k^2 = \sum\left(H_k^2 - V_k^2\right) = 2^2 \sum \cos\left((2\,k-1)\,\frac{2\,\pi}{n}\right).$$

But the points $\left(\cos\left((2\,k-1)\,\frac{2\,\pi}{n}\right), \sin\left((2\,k-1)\,\frac{2\,\pi}{n}\right)\right)$ form a regular $\frac{n}{2}$-gon, so they sum to $(0, 0)$ and the cosines therefore sum to 0. □